

SOUND WAVES

In Chapter 18 we studied transverse mechanical waves, in particular the vibrations of a stretched string. Now we turn our attention to longitudinal mechanical waves, in particular sound waves. What we call sound is a longitudinal mechanical vibration with frequencies from about 20 Hz to about 20,000 Hz, which is the typical range of human hearing. Longitudinal waves of higher frequency, which are called ultrasonic waves, are used in locating underwater objects and in medical imaging. Longitudinal (and transverse) mechanical waves of lower frequency, called infrasonic, occur as seismic waves in earthquakes.

In this chapter we discuss the properties of sound waves, their propagation, and their production by vibrating systems.

19-1 PROPERTIES OF SOUND WAVES

Like the transverse wave on the string, sound is a mechanical wave, meaning that the disturbance propagates due to the mechanical (elastic) forces between particles in the medium. Mechanical waves can travel through any material medium (solid, liquid, or gas). In solids, mechanical waves can be longitudinal or transverse, but in fluids (which cannot support shearing forces) the waves are only longitudinal, which means that the particles of the medium oscillate along the same direction that the wave is traveling.

When we discuss sound waves, we normally mean longitudinal waves in the frequency range 20 Hz to 20,000 Hz, the normal range of human hearing. However, the branch of physics and engineering that deals with the study of sound waves, called *acoustics*, generally includes the study of mechanical waves of all frequencies, with transverse as well as longitudinal vibrations in the case of solids. In this chapter we consider mainly sound waves in air, which are strictly longitudinal.

Although a small source of sound in an open area emits waves that are three-dimensional, we will simplify the prob-

lem by considering one-dimensional waves. Figure 19-1 shows how a one-dimensional sound wave might be established in a long tube filled with air. At one end of the tube is a piston, which might represent the moving cone of a loud-

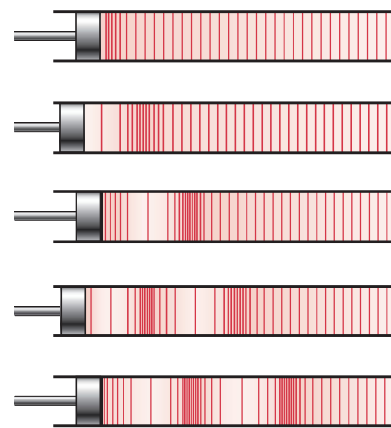


FIGURE 19-1. Sound waves generated in a tube by a moving piston, which might represent the moving cone of a loudspeaker. The vertical lines divide the compressible medium in the tube into layers of equal mass.

speaker. As the piston moves back and forth, it alternately compresses and expands (rarefies) the air next to it. This disturbance travels down the tube as a sound wave. As the wave passes any point, air molecules at that location move back and forth about their equilibrium positions parallel to the direction that the wave is traveling.

As we will see, we can describe the sound wave either in terms of changes in the local pressure in the medium or else in terms of the displacement of the air molecules from their equilibrium positions. These two descriptions convey the same information, but they have somewhat different mathematical forms.

19-2 TRAVELING SOUND WAVES

As the piston in Fig. 19-1 oscillates, it causes variations in the density of the air in the tube from one place to another and also from one instant of time to another, as Fig. 19-1 shows. The regions of high density are called *compressions*, and the regions of low density are called *rarefactions*. As the sound wave travels, the compressions and rarefactions travel along the tube.

We express the density in the tube as $\rho(x, t)$, a function both of location and time. The undisturbed density of the air in the tube is ρ_0 , and the sound wave causes fluctuations in the density $\Delta\rho(x, t)$ that are very small compared with ρ_0 . That is $\rho(x, t) = \rho_0 + \Delta\rho(x, t)$, where $\Delta\rho(x, t)$ can be positive or negative but $|\Delta\rho(x, t)| \ll \rho_0$.

We can also describe the sound wave in terms of the variation in the pressure in the tube. The pressure variations travel along the tube in phase with the density variations: when the density reaches its maximum at a particular location, the pressure also reaches its maximum at that location. The undisturbed pressure in the absence of the sound wave is p_0 , and the pressure fluctuations $\Delta p(x, t)$ can be positive or negative but are very small in comparison with p_0 . The total pressure at the point with coordinate x and at time t is $p(x, t) = p_0 + \Delta p(x, t)$.

Let us assume that the piston is driven so that its position can be described by a sine or cosine function; then the density and pressure of air in the tube will also vary sinusoidally. Figure 19-2a shows a “snapshot” of the density of the air in the tube at a particular instant of time, and Figs. 19-2b and c show the corresponding variations in the density and pressure of the air. The density fluctuates with amplitude $\Delta\rho_m$ about the value ρ_0 , and the pressure with amplitude Δp_m about p_0 . For sinusoidal waves, neglecting for now any phase constant, we can write

$$\Delta\rho(x, t) = \Delta\rho_m \sin(kx - \omega t). \quad (19-1)$$

The pressure variations can be written similarly as

$$\Delta p(x, t) = \Delta p_m \sin(kx - \omega t). \quad (19-2)$$

Usually it is most convenient to describe the sound wave in terms of its pressure variation.

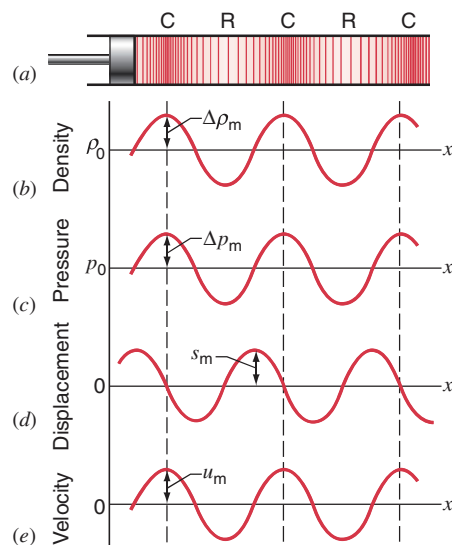


FIGURE 19-2. (a) Compressions C and rarefactions R in a sound wave traveling along a tube. (b) A snapshot of the density, which varies with amplitude $\Delta\rho_m$ about the value ρ_0 . (c) A snapshot of the pressure, which varies with amplitude Δp_m about p_0 . (d) The longitudinal displacement, which shows at every location x how a small element of air has been displaced from its equilibrium position. (e) The longitudinal velocity of the small elements of air.

The relationship between the pressure amplitude Δp_m and the density amplitude $\Delta\rho_m$ depends on the mechanical properties of the medium. In Eq. 15-5 we introduced the bulk modulus $B = -\Delta p/(\Delta V/V)$, which describes the relative change in volume of an element of fluid in response to a change in pressure. With $\rho = m/V$, we have $d\rho = -(m/V^2)dV = -(\rho/V)dV$. Replacing the differentials with differences, we can write $\Delta\rho = -\rho(\Delta V/V)$. Using Eq. 15-5 to replace $\Delta V/V$ with $-\Delta p/B$, and replacing ρ with its undisturbed value ρ_0 (because the fluctuations $\Delta\rho$ are small), we have

$$\Delta\rho = -\rho \frac{\Delta V}{V} = -\rho_0 \frac{-\Delta p}{B} = \Delta p \frac{\rho_0}{B}$$

or, in terms of the density and pressure amplitudes,

$$\Delta\rho_m = \Delta p_m \frac{\rho_0}{B}. \quad (19-3)$$

Because Eq. 15-5 applies to all fluids (not only to air), Eq. 19-3 similarly applies to sound waves in all fluids.

There is one special caution that must be observed in using Eq. 19-3. As a sound wave moves along the tube, it alternately compresses and expands each small volume of air. When a volume element of gas is compressed, work is done on it, so that its internal energy and its temperature increase. In fluids, the rate at which energy can flow (as heat) from one volume element to another is generally rather small, so that the internal energy increase associated with

the compression does not have sufficient time at sound-wave frequencies to be transferred as heat to the adjacent cooler regions associated with the rarefactions. We call this an *adiabatic* (no heat transfer) process, and in Eq. 19-3 we must therefore use the adiabatic bulk modulus. Otherwise, if heat could flow so that the adjacent regions could reach a common temperature, we would use the *isothermal* (constant temperature) bulk modulus. For a gas such as air, the adiabatic bulk modulus is about 1.4 times the isothermal bulk modulus. Isothermal and adiabatic processes are considered in greater detail in Chapter 23.

Sound as a Displacement Wave

Just as we considered the displacement and velocity of “particles” of the medium for transverse mechanical waves on a string in Chapter 18, it is also instructive to consider the displacement and velocity associated with the longitudinal sound wave. In this case, “particles” of the medium refer to elements of volume of the fluid.

Figure 19-3 shows an element of gas inside the tube through which the sound wave is traveling. The element has mass δm , and in the equilibrium state it is located at the coordinate x and has thickness δx and cross-sectional area A . The undisturbed density of this fluid element is

$$\rho_0 = \frac{\delta m}{A \delta x}. \quad (19-4)$$

As the sound wave passes, this element of fluid oscillates about its equilibrium position. We represent the displacement of the fluid element from this equilibrium position by the function $s(x, t)$. The value of this function varies from place to place and also from time to time, as different fluid elements are moved in different directions by the sound wave. The function $s(x, t)$ is similar to the transverse wave function $y(x, t)$, with one important exception: the displacement s is measured along the direction of propagation for a longitudinal wave, instead of at right angles to the direction of propagation (in the case of the transverse wave of Chapter 18).

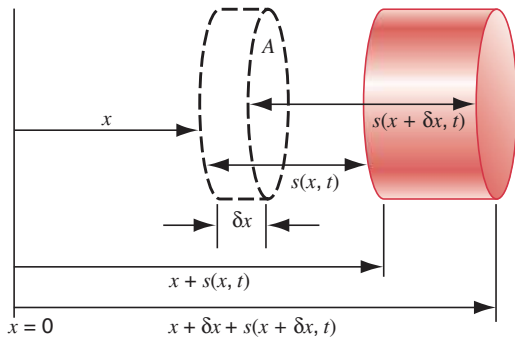


FIGURE 19-3. An element of air thickness δx and cross-sectional area A is originally located at x . The sound wave displaces the element by an amount $s(x, t)$.

In response to the sound wave, the left-hand edge of the fluid element moves from coordinate x to coordinate $x + s(x, t)$, as shown in Fig. 19-3. The right-hand edge of the element moves from coordinate $x + \delta x$ to $x + \delta x + s(x + \delta x, t)$. That is, in response to the sound wave, the fluid element not only moves, but its thickness changes from δx to $[x + \delta x + s(x + \delta x, t)] - [x + s(x, t)] = \delta x + s(x + \delta x, t) - s(x, t)$, which we can write as

$$\delta x \left[1 + \frac{s(x + \delta x, t) - s(x, t)}{\delta x} \right].$$

In the limit of a very thin volume element, in which $\delta x \rightarrow 0$, the quotient in this expression can be written as $\partial s / \partial x$, the partial derivative of s with respect to x , so that the thickness of the element is $\delta x [1 + \partial s / \partial x]$. The density of this fluid element is now

$$\rho = \frac{\delta m}{A \delta x (1 + \partial s / \partial x)} = \frac{\rho_0}{1 + \partial s / \partial x} \quad (19-5)$$

using Eq. 19-4 for ρ_0 . We have assumed that the density variations are very small, so that $\partial s / \partial x \ll 1$. Using the binomial expansion (see Appendix I), $(1 + z)^{-1} = 1 - z + \dots$, we keep only the first term in the expansion and obtain $\rho = \rho_0 (1 - \partial s / \partial x)$. The change in density is $\Delta \rho(x, t) = \rho - \rho_0$, or

$$\Delta \rho(x, t) = -\rho_0 \frac{\partial s}{\partial x}. \quad (19-6)$$

For sinusoidal waves, we can use Eq. 19-1 for $\Delta \rho(x, t)$, and so obtain

$$\frac{\partial s}{\partial x} = -\frac{\Delta \rho(x, t)}{\rho_0} = -\frac{\Delta \rho_m}{\rho_0} \sin(kx - \omega t).$$

Integrating with respect to x gives

$$s(x, t) = \frac{\Delta \rho_m}{k \rho_0} \cos(kx - \omega t) = s_m \cos(kx - \omega t). \quad (19-7)$$

The displacement amplitude is

$$s_m = \frac{\Delta \rho_m}{k \rho_0} = \frac{\Delta p_m}{kB}, \quad (19-8)$$

where we have used Eq. 19-3 to relate $\Delta \rho_m$ to Δp_m .

Note that the displacement variation is expressed as a cosine function when the pressure and density variations are expressed as sine functions. Thus the displacement variations are 90° out of phase with the pressure and density variations. The displacement waveform is shown in Fig. 19-2d. At locations where the pressure and density are a maximum or minimum, the displacement of the elements of fluid is zero; the displacement is a maximum or minimum where the pressure and density variations are zero. For example, at a compression, fluid elements to the left must have positive displacements and fluid elements to the right must have negative displacements, consistent with fluid elements at the center of the compression having zero displacements.

As a fluid element oscillates about its equilibrium position, its longitudinal velocity is

$$u_x(x, t) = \frac{\partial s}{\partial t} = \omega s_m \sin(kx - \omega t) = u_m \sin(kx - \omega t), \quad (19-9)$$

where the amplitude of the velocity variations is

$$u_m = \omega s_m = \frac{\omega \Delta p_m}{kB} = \frac{v \Delta p_m}{B} \quad (19-10)$$

using Eq. 19-8 and Eq. 18-13 ($v = \omega/k$). Once again, it is important to note that this is a longitudinal velocity, directed along the axis of the tube (parallel to the direction of propagation of the wave). The velocity variations are in phase with the pressure and density variations, as shown in Fig. 19-2e.

Although we have described a sound wave in terms of either a pressure wave or a displacement wave, the two descriptions are in general not equivalent. Only when a single longitudinal wave is propagating in a single direction can we easily shift back and forth between the two descriptions. When we consider the reflection of a sound wave at the end of a tube, or when we superimpose two sound waves that interfere at a point, using the displacement wave description can lead to serious errors.* For example, consider two sound waves from different sources (two loudspeakers, for instance) that travel along different directions and interfere at a point, such that one wave gives a pressure change Δp and the other $-\Delta p$. On the basis of the pressure description, we expect complete destructive interference at that point, because the pressures add like scalars. However, the displacements (which are along the directions of travel of the two waves) do not add to zero, because they are vectors in different directions. *It is usually preferable to describe a sound wave as a pressure wave* to avoid such difficulties. Moreover, as we shall see in the next section, it is the pressure change, not the displacement, that is detected by ears and microphones.

SAMPLE PROBLEM 19-1. The maximum pressure variation Δp_m that the ear can tolerate in loud sounds is about 28 Pa at 1000 Hz. The faintest sound that can be heard at 1000 Hz has a pressure amplitude of about 2.8×10^{-5} Pa. Find the corresponding density and displacement amplitudes. The bulk modulus for air under standard conditions is 1.4×10^5 Pa, and the speed of sound in air is 343 m/s at room temperature.

Solution The wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} = \frac{2\pi \times 10^3 \text{ Hz}}{343 \text{ m/s}} = 18.3 \text{ rad/m}.$$

* For a careful discussion of this point, see “Pressure and Displacement in Sound Waves,” by C. T. Tindle, *American Journal of Physics*, September 1984, p. 749.

The density of air under these conditions is 1.21 kg/m^3 . Hence for $\Delta p_m = 28 \text{ Pa}$, we obtain, using Eq. 19-3,

$$\begin{aligned} \Delta \rho_m &= \Delta p_m \frac{\rho_0}{B} = (28 \text{ Pa}) \frac{1.21 \text{ kg/m}^3}{1.4 \times 10^5 \text{ Pa}} \\ &= 2.4 \times 10^{-4} \text{ kg/m}^3 \end{aligned}$$

and, using Eq. 19-8,

$$\begin{aligned} s_m &= \frac{\Delta p_m}{kB} = \frac{28 \text{ Pa}}{(18.3 \text{ rad/m})(1.4 \times 10^5 \text{ Pa})} \\ &= 1.1 \times 10^{-5} \text{ m}. \end{aligned}$$

The displacement amplitudes for the loudest sounds are about 10^{-5} m , a very small value indeed. For the faintest sounds, we have similarly

$$\begin{aligned} \Delta \rho_m &= (2.8 \times 10^{-5} \text{ Pa}) \frac{1.21 \text{ kg/m}^3}{1.4 \times 10^5 \text{ Pa}} \\ &= 2.4 \times 10^{-10} \text{ kg/m}^3 \end{aligned}$$

and

$$s_m = \frac{2.8 \times 10^{-5} \text{ Pa}}{(18.3 \text{ rad/m})(1.4 \times 10^5 \text{ Pa})} = 1.1 \times 10^{-11} \text{ m}.$$

This is about one-tenth of the radius of a typical atom and suggests how sensitive the ear must be to detect vibrations of such a small amplitude.

19-3 THE SPEED OF SOUND

As in the case of the transverse mechanical wave, the speed of a sound wave depends on the ratio of an elastic property of the medium and an inertial property. In analogy with Section 18-4, we might guess that the elastic property is the undisturbed pressure and the inertial property is the undisturbed density and try to do a dimensional analysis based on

$$v \propto \frac{p_0^a}{\rho_0^b}$$

to determine the exponents a and b . The dimensional analysis gives $a = b = \frac{1}{2}$, as you should verify. However, in this case we are not as fortunate as we were in the case of transverse waves—the expression $v = (p_0/\rho_0)^{1/2}$ does not give the correct value for the speed of sound. For example, in air at 20°C we would obtain $v = 289 \text{ m/s}$ from this expression, which is in poor agreement with the measured value, $v = 343 \text{ m/s}$. You will recall that dimensional analysis gives us only the functional dependence and is unable to provide the values of any dimensionless constants that may be part of the equation. In this case we are clearly missing an important constant.

Let us turn instead to a mechanical analysis. Rather than a sinusoidal wave, it is simpler to consider only a single compressional pulse that is traveling down the tube. The pulse is traveling at speed v , the wave speed. Within the

pulse, which has a width L , the pressure increase is a constant Δp .

Figure 19-4a shows the pulse as it is just about to enter a fluid element of length L_0 and cross-sectional area A where the undisturbed pressure is p_0 . After a time t , the leading edge of the pulse has reached the end of the fluid element, which has now been compressed into a length L (Fig. 19-4b).

During the time t two horizontal forces act on the element: the compressional pulse exerts a force $(p_0 + \Delta p)A$ to the right, while the undisturbed fluid on the right of the element exerts a force p_0A that acts to the left. The net external force on the element during this interval is $A \Delta p$, acting to the right.

We cannot apply the particle form of Newton's laws to this element, because it does not behave like a particle; all parts of the element do not move in the same way. Instead, we can treat the element as a system of particles and use Newton's law in the form of Eq. 7-17 ($\Sigma F_{\text{ext},x} = Ma_{\text{cm},x}$). Measured from the right-hand end of the element, the center of mass moves from a location $-L_0/2$ to $-L/2$ in a time t . The acceleration of the center of mass is, using Eq. 2-28 in the form $x - x_0 = \frac{1}{2}a_x t^2$,

$$a_{\text{cm},x} = \frac{2(x - x_0)}{t^2} = \frac{2[(-L/2) - (-L_0/2)]}{t^2} = -\frac{\Delta L}{t^2}, \quad (19-11)$$

where $\Delta L = L - L_0$ is the change in length of the fluid element. The mass of the fluid element is $M = \rho_0 A L_0$ and so Newton's second law gives

$$A \Delta p = (\rho_0 A L_0)(-\Delta L/t^2). \quad (19-12)$$

If the pulse moves with speed v , the time needed for it to move through the fluid element is $t = L_0/v$. Making this substitution into Eq. 19-12 and rearranging terms, we obtain

$$v^2 = \frac{1}{\rho_0} \frac{-\Delta p}{A \Delta L / A L_0}. \quad (19-13)$$

The quantity $A \Delta L$ is the change in volume ΔV of the fluid element, and $A L_0$ is its original volume V . With these substitutions, the second factor on the right of Eq. 19-13 be-

comes $-\Delta p/(\Delta V/V)$, which is just the bulk modulus B . Taking the square root on both sides, we can thus write Eq. 19-13 as

$$v = \sqrt{\frac{B}{\rho_0}}. \quad (19-14)$$

Equation 19-14 gives the speed of sound in fluids in terms of the bulk modulus and the density. Note that, as in the case of transverse waves on a string, the speed of sound in a fluid depends only on the properties of the medium and not on the frequency or wavelength of the wave.

In gases, the bulk modulus can be written γp_0 , where γ is a constant (called the *specific heat ratio* and discussed in Chapter 23) that depends on the type of gas and typically has values between 1.3 and 1.7. For air, $\gamma = 1.4$; this factor accounts for the discrepancy in the result for the speed of sound obtained from the dimensional analysis at the beginning of this section.

Equation 19-14 applies to fluids (gases and liquids) but not to solids. In solids, a shearing modulus as well as a compressional modulus may be present, and the analysis is often more complicated than the simple one-dimensional case presented here.

Table 19-1 gives some representative values for the speed of sound in various materials.

Finally, we note that in this section we have treated the fluid as a continuous medium. In a gas, however, the spaces between molecules are large (compared with the size of the molecules), and the molecules move with a random thermal motion. The oscillations produced by a sound wave are superimposed on these random thermal motions. An impulse given to one molecule is passed on to another molecule only after the first has moved through the empty space between them and collided with the second. There is thus a close connection between the average molecular speed in a fluid and the speed of sound in that fluid. In particular, as we in-

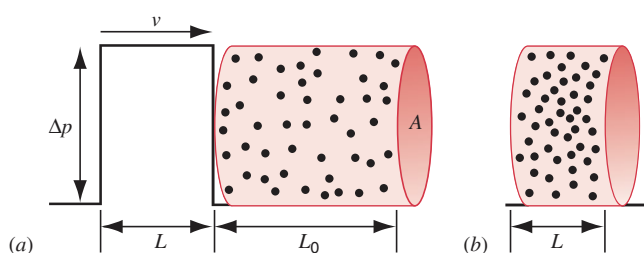


FIGURE 19-4. (a) A compressional pulse is about to enter a fluid element of undisturbed length L_0 . (b) A time t later, the pulse has reached the right-hand end of the element and compressed the element to a length L .

TABLE 19-1 The Speed of Sound^a

Medium	Speed (m/s)
Gases	
Air (0°C)	331
Air (20°C)	343
Helium	965
Hydrogen	1284
Liquids	
Water (0°C)	1402
Water (20°C)	1482
Seawater ^b	1522
Solids^c	
Aluminum	6420
Steel	5941
Granite	6000

^a At 0°C and 1 atm pressure, unless indicated otherwise.

^b At 20°C and 3.5% salinity.

^c Longitudinal waves; speeds of transverse waves are about half as large as those for longitudinal waves.

crease the temperature, the average molecular speed and the speed of sound in a gas increase in exactly the same manner.

19-4 POWER AND INTENSITY OF SOUND WAVES

In Section 18-6 we found the intensity of transverse waves on a string by considering the kinetic and potential energies associated with the wave motion. We can do the same type of analysis for sound waves by calculating the kinetic energy due to the motion of each fluid element and the internal energy stored in each fluid element (which is very similar in form to potential energy) as the wave passes. However, the internal energy calculation requires details of the dynamic behavior of gases that we shall not discuss until Chapter 23. As a result, we will use a different method to find the power transferred by a sound wave.

As the wave travels, each fluid element exerts a force on the fluid element ahead of it. If the pressure increase in the fluid element is Δp , the force it exerts on the next element is $F_x = A \Delta p$, where A is the cross-sectional area of the fluid element. Using Eq. 19-2 for the pressure, we find that the force is

$$F_x = A \Delta p_m \sin(kx - \omega t). \quad (19-15)$$

The velocity u_x of the thin slice of fluid is given by Eq. 19-9. The power delivered to the fluid element is

$$P = u_x F_x = A \Delta p_m u_m \sin^2(kx - \omega t). \quad (19-16)$$

Using Eq. 19-10 we can write this as

$$P = \frac{A v (\Delta p_m)^2}{B} \sin^2(kx - \omega t). \quad (19-17)$$

As we did in Section 18-6, we assume that we observe the waves during a time that is very long compared with the period of oscillation of the wave. Since the average value of $\sin^2 \theta$ over any number of full cycles is $\frac{1}{2}$, the average power is

$$P_{\text{av}} = \frac{A v (\Delta p_m)^2}{2B} = \frac{A (\Delta p_m)^2}{2\rho v} \quad (19-18)$$

using Eq. 19-14 to replace B with ρv^2 .

As in the case of the transverse wave, the power depends on the *square* of the amplitude, in this case the pressure amplitude. Note also that the frequency does not appear explicitly in Eq. 19-18 (although it would appear if we instead expressed the average power in terms of the displacement amplitude). Hence, by measuring pressure amplitudes, we can directly compare the intensities of sounds having *different* frequencies. For this reason, instruments that measure pressure changes are preferable to those that measure displacements; moreover, as we learned from Sample Problem 19-1, displacements from the weakest audible sounds are very small and would be difficult to measure directly.

When we are comparing different sounds, it is more useful to use the *intensity* (average power per unit area) of

the wave. From Eq. 19-18, we can immediately obtain the intensity I :

$$I = \frac{P_{\text{av}}}{A} = \frac{(\Delta p_m)^2}{2\rho v}. \quad (19-19)$$

The response of the ear to sound of increasing intensity is approximately logarithmic, so it is convenient to introduce a logarithmic scale of intensity called the *sound level* SL :

$$SL = 10 \log \frac{I}{I_0}. \quad (19-20)$$

The SL is defined with respect to a reference intensity I_0 , which is chosen to be 10^{-12} W/m^2 (a typical value for the threshold of human hearing). Sound levels defined in this way are measured in units of *decibels* (dB). A sound of intensity I_0 has a sound level of 0 dB, whereas sound at the upper range of human hearing, called the threshold of pain, has an intensity of 1 W/m^2 and a SL of 120 dB. Multiplication of the intensity I by a factor of 10 corresponds to adding 10 dB to the SL .

We can also use dB as a *relative* measure to compare different sounds with one another, rather than with the reference intensity. Suppose we wish to compare two sounds of intensities I_1 and I_2 :

$$\begin{aligned} SL_1 - SL_2 &= 10 \log \frac{I_1}{I_0} - 10 \log \frac{I_2}{I_0} \\ &= 10 \log \frac{I_1}{I_2}. \end{aligned} \quad (19-21)$$

For example, two sounds whose intensity ratio is 2 differ in SL by $10 \log 2 = 3 \text{ dB}$.

The sensitivity of the human ear varies with frequency. The threshold of 10^{-12} W/m^2 applies only to the midrange

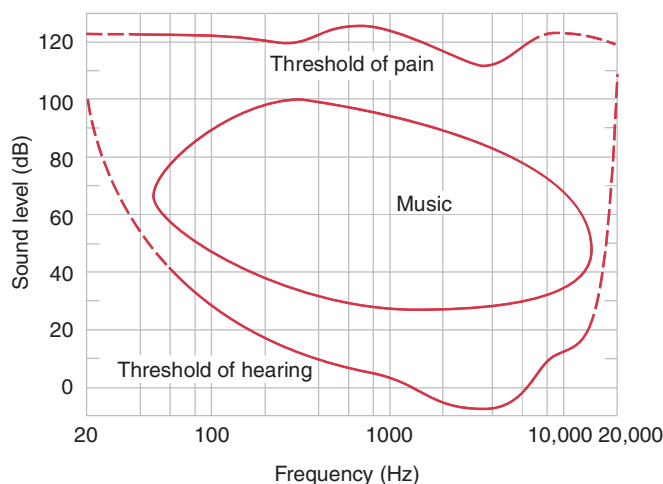


FIGURE 19-5. The average range of sound levels for human hearing. Note the dependence of the threshold levels on frequency. A sound that we can just hear at 100 Hz must have 1000 times the acoustic power (30 dB greater sound level) than one we can just hear at 1000 Hz, because our ear is that much less sensitive at 100 Hz.

TABLE 19-2 Some Intensities and Sound Levels

Sound	Intensity I (W/m^2)	Sound Level (dB)
Threshold of hearing	1×10^{-12}	0
Rustle of leaves	1×10^{-11}	10
Whisper (at 1 m)	1×10^{-10}	20
City street, no traffic	1×10^{-9}	30
Office, classroom	1×10^{-7}	50
Normal conversation (at 1 m)	1×10^{-6}	60
Jackhammer (at 1 m)	1×10^{-3}	90
Rock group	1×10^{-1}	110
Threshold of pain	1	120
Jet engine (at 50 m)	10	130
Space shuttle engine (at 50 m)	1×10^8	200

frequencies around 1000 Hz. At the higher frequencies, say 10,000 Hz, the threshold rises to about 10 dB (10^{-11} W/m^2), whereas at a lower frequency of 100 Hz the threshold is about 30 dB (10^{-9} W/m^2). It takes 1000 times the sound intensity at 100 Hz to produce the same physiological response as a given sound intensity at 1000 Hz. Figure 19-5 shows the variation with frequency of the thresholds of hearing and of pain, and Table 19-2 shows some representative sound levels and their corresponding intensities.

SAMPLE PROBLEM 19-2. Spherical sound waves are emitted uniformly in all directions from a point source, the radiated power P being 25 W. What are the intensity and the sound level of the sound wave a distance $r = 2.5 \text{ m}$ from the source?

Solution All the radiated power P must pass through a sphere of radius r centered on the source. Thus

$$I = \frac{P}{4\pi r^2}.$$

We see that the intensity of the sound drops off as the inverse square of the distance from the source. Numerically, we have

$$I = \frac{25 \text{ W}}{(4\pi)(2.5 \text{ m})^2} = 0.32 \text{ W/m}^2$$

and

$$\begin{aligned} SL &= 10 \log \frac{I}{I_0} \\ &= 10 \log \frac{0.32 \text{ W/m}^2}{10^{-12} \text{ W/m}^2} = 115 \text{ dB}. \end{aligned}$$

A comparison of this result with the list in Table 19-2 shows this sound level to be dangerous to a person's hearing.

19-5 INTERFERENCE OF SOUND WAVES

In Section 18-8 we discussed the interference that can occur when two different waves exist at the same point in space at the same time. The principle of superposition,

which we used to add transverse waves on a string, also applies to sound waves.

Figure 19-6 shows two loudspeakers driven from a common source. At point P , the pressure variation due only to speaker S_1 is Δp_1 , and that due to S_2 alone is Δp_2 . (Note that these are spherical waves and are therefore not described by Eq. 19-2; in particular, the pressure amplitude does not remain constant as a spherical wave travels but instead decreases like $1/r$). The total pressure disturbance at point P is $\Delta p = \Delta p_1 + \Delta p_2$.

The type of interference that occurs at point P depends on the *phase difference* $\Delta\phi$ between the waves. In contrast to Section 18-8, in which the phase difference between the waves was due to their different phase constants, in this case the phase difference comes about because the waves may travel different distances from the speakers to arrive at point P . The phase difference $\Delta\phi$ between the two waves arriving at P depends on the path difference $\Delta L = |r_1 - r_2|$ from the speakers to point P . Path difference and phase difference are related by

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta L}{\lambda}. \quad (19-22)$$

That is, a phase difference of π corresponds to a path difference of $\lambda/2$, a phase difference of 2π corresponds to a path difference of λ , and so forth.

For some locations of point P , the pressure variations arrive in phase ($\Delta\phi = 0, 2\pi, 4\pi, \dots$) and interfere constructively. For other locations of P , the waves arrive out of phase ($\Delta\phi = \pi, 3\pi, 5\pi, \dots$) and interfere destructively. Using the condition for constructive interference, $\Delta\phi = m(2\pi)$ with $m = 0, 1, 2, \dots$, Eq. 19-22 shows that the corresponding path difference for constructive interference is

$$\Delta L = m\lambda \quad (m = 0, 1, 2, \dots). \quad (19-23)$$

That is, at locations where $|r_1 - r_2| = 0, \lambda, 2\lambda, \dots$, the intensity reaches a maximum value. If the speakers are driven

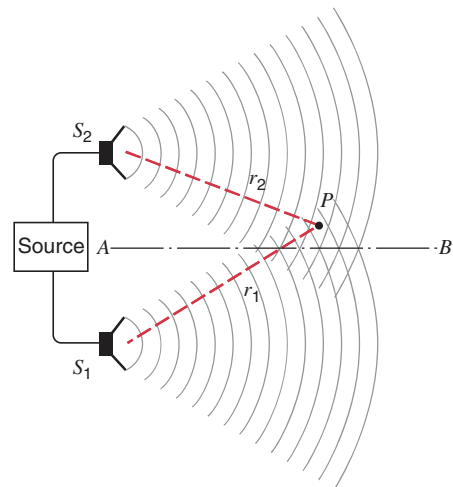


FIGURE 19-6. Two loudspeakers S_1 and S_2 , driven by a common source, send signals to point P , where the signals interfere.

ven in phase, then at all points equidistant from the speakers (the line AB , which represents the entire midplane) there is constructive interference.

For destructive interference, the phase difference is $\Delta\phi = (m + \frac{1}{2})2\pi$ with $m = 0, 1, 2, \dots$, so the path difference for destructive interference is

$$\Delta L = (m + \frac{1}{2})\lambda \quad (m = 0, 1, 2, \dots). \quad (19-24)$$

That is, at locations where $|r_1 - r_2| = \lambda/2, 3\lambda/2, 5\lambda/2, \dots$, the intensity has a minimum value (not necessarily zero, because in general the two waves arrive at point P with difference amplitudes). Locations of destructive interference correspond to “dead spots” in the listening environment of the speakers.

If the speakers emit a mixture of many different wavelengths, some points P might show destructive interference for one wavelength and constructive interference for another.

SAMPLE PROBLEM 19-3. In the geometry of Fig. 19-6, a listener is seated at a point a distance of 1.2 m directly in front of one speaker. The two speakers, which are separated by a distance D of 2.3 m, emit pure tones of wavelength λ . The waves are in phase when they leave the speakers. For what wavelengths will the listener hear a minimum in the sound intensity?

Solution The minimum sound intensity occurs when the waves from the two speakers interfere destructively, according to the criteria of Eq. 19-24. If the listener is seated in front of speaker 2, then $r_2 = 1.2$ m, and r_1 can be found from the Pythagorean formula,

$$r_1 = \sqrt{r_2^2 + D^2} = \sqrt{(1.2 \text{ m})^2 + (2.3 \text{ m})^2} = 2.6 \text{ m}.$$

Thus $r_1 - r_2 = 2.6 \text{ m} - 1.2 \text{ m} = 1.4 \text{ m}$, and, according to Eq. 19-24, we have

$$1.4 \text{ m} = \lambda/2, 3\lambda/2, 5\lambda/2, \dots,$$

corresponding to

$$\lambda = 2.8 \text{ m}, 0.93 \text{ m}, 0.56 \text{ m}, \dots$$

Complete destructive interference will not occur at this location, because the two waves arriving at the observation point have different amplitudes, if they leave the speakers with equal amplitudes.

19-6 STANDING LONGITUDINAL WAVES

We now consider what happens when a sound wave such as that shown in Fig. 19-1 reaches the end of the tube. In analogy with the transverse wave on the string (see Fig. 18-19), a reflection occurs, and the reflected wave travels back down the tube in the opposite direction. The behavior of the wave at the reflecting end depends on whether the end of the tube is open or closed.

Let us first consider a tube that is closed at the end. As the wave travels down the tube and reaches the end, it can

compress the layers of air at the closed end against the fixed barrier. At that end, the pressure can therefore vary with its maximum amplitude, and *the closed end is a pressure antinode*. A pressure wave reflects from a closed end in a manner similar to the reflection of a transverse displacement wave at the free end of a string (Fig. 18-19b). If a compression, for example, is incident on the closed end, it reflects back along the tube as a compression. In analogy with our discussion of transverse waves on strings, we say that *a longitudinal pressure wave is reflected from a closed end with no change of phase*. The same effect occurs in the case of a longitudinal wave traveling on a spring such as a Slinky toy and reflecting from a *fixed* end: a compression is reflected as a compression.

Now consider what happens if the end of the tube is open. The pressure at the end of the open tube is the same as the ambient pressure p_0 in the surrounding room. We cannot change the pressure at that end of the tube unless we change the pressure in the entire room. The pressure at the open end therefore remains at the value p_0 , and *the open end is a pressure node*. Comparison with Fig. 18-19a shows that this case is analogous to the transverse displacement wave reflecting from the fixed end of the string. The attempt by the wave incident on the open end to compress the air at that end causes a rarefaction, which travels back down the tube in the opposite direction. Thus *a longitudinal pressure wave is reflected from an open end with a phase change of 180°* . The same effect can again be observed with a coiled spring: a compression is reflected as a rarefaction.

Let us now assume that we have a train of sinusoidal waves traveling down the tube. The waves are reflected at the end, which will behave either as a pressure node (if the end is open) or a pressure antinode (if the end is closed). We assume the source of the wavetrain is a speaker at the opposite end. The movement of the speaker sends a compressional wave down the tube, and the superposition of the original and reflected waves gives a pattern of standing waves, just as was the case for the transverse waves on the string. Within the tube will be a pattern of pressure nodes and antinodes (which are not points, as in the case of transverse waves on a string, but planes).

If the frequency (or wavelength) of the source of the waves is selected to be a particular value that depends on the length of the tube, then a standing wave pattern is established along the entire tube, in analogy with the case of the standing wave patterns shown in Fig. 18-20. If there is a node of pressure at the speaker end, then little energy is given back to the speaker from the standing wave pattern in the tube, and we have a condition of resonance. The driving frequency must be equal to one of the natural frequencies of the system, which are determined by the length of the tube.

Figure 19-7a shows a tube that is driven by a speaker at one end and is open at the other end. As we have discussed previously, the speaker end is a pressure node at resonance and the open end is likewise a pressure node. In Fig. 19-7a

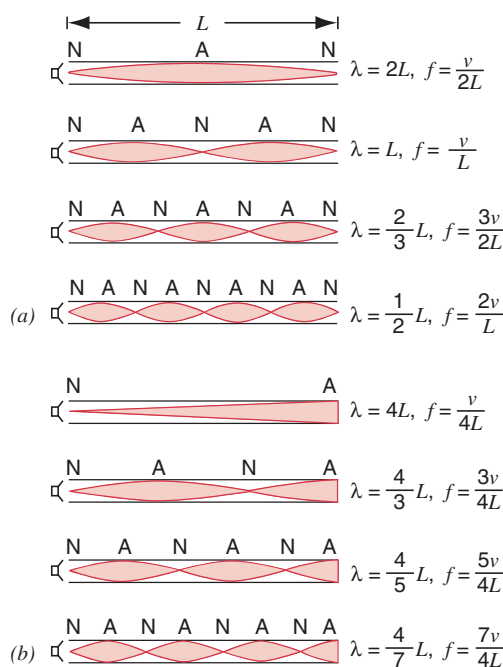


FIGURE 19-7. (a) The pressure waves of the first four resonant modes of a tube driven by a speaker and open at the other end. There is a pressure node N at each end, and antinodes A are located between the nodes. The curves suggest the sinusoidal variation of pressure within the tube. (b) The pressure waves of the first four resonant modes of a tube that is closed at one end. The closed end is a pressure antinode. Note the differences in vibrational patterns and wavelengths between the open and closed tubes.

are shown the resulting variations in the pressure amplitude of the standing waves.* These patterns look very similar to those of Fig. 18-20. In the first mode of oscillation, the length L of the tube is equal to $\lambda/2$, where λ is the wavelength of the wave produced by the speaker for that particular resonant condition. The wavelength is therefore $2L$, and the corresponding frequency is $f = v/\lambda = v/2L$. The other resonances shown in Fig. 19-7a have successively smaller wavelengths, which can be written in general as

$$\lambda_n = \frac{2L}{n}, \quad n = 1, 2, 3, \dots \quad (19-25)$$

The corresponding resonant frequencies, determined using the expression $f = v/\lambda$ with the above wavelengths, are

$$f_n = n \frac{v}{2L}, \quad n = 1, 2, 3, \dots \quad (\text{open tube}). \quad (19-26)$$

Here v represents the speed of the wave in the medium that fills the tube, usually air.

* A beautiful demonstration of the locations of the pressure nodes and antinodes can be obtained with a Rubens flame-tube. See "Rubens Flame-tube Demonstration," by George W. Ficken and Francis C. Stephenson, *The Physics Teacher*, May 1979, p. 306.

Figure 19-7b shows the case in which the tube is closed at one end and open at the other end. In this case, the closed end must be a pressure antinode. In the first resonant mode, the length L of the tube is $\frac{1}{4}\lambda$, and so the source must be producing a wave whose wavelength is $4L$. In the next mode, the wavelength changes so that now L is $\frac{3}{4}\lambda$, and thus $\lambda = \frac{4}{3}L$. Continuing the series, we see that in this case the general expression for the wavelengths of the resonant modes is

$$\lambda_n = \frac{4L}{n}, \quad n = 1, 3, 5, \dots \quad (19-27)$$

Note that only odd values of the integer n appear in this case. The corresponding resonant frequencies are

$$f_n = n \frac{v}{4L}, \quad n = 1, 3, 5, \dots \quad (\text{closed tube}). \quad (19-28)$$

As we discuss in the next section, the resonant frequencies given by Eq. 19-26 or 19-28 determine the musical notes played by the wind instruments.

The actual location of the pressure node at an open end is not *exactly* at the end of the tube. The wave extends slightly into the medium beyond the tube, so the effective length of the tube is a bit greater than its actual length and the resonant frequencies are a bit smaller. For narrow tubes of cylindrical shape, the length correction is roughly equal to $0.6R$, where R is the radius of the tube. For a tube open at both ends, the length correction must be applied at each end. For a tube of length 0.6 m and radius 1 cm (typical values for the smaller wind instruments such as the clarinet or flute), the lowest frequency without the end correction would be 286 Hz if the tube were open and 143 Hz if the tube were closed. With the end correction, the corresponding values would be 280 Hz and 142 Hz. The corrections are small, but nevertheless quite important.

SAMPLE PROBLEM 19-4. Figure 19-8 shows an apparatus that can be used to measure the speed of sound in air by using the resonance condition. A small speaker is held above a cylindrical tube partially filled with water. By adjusting the water level, the length of the air column can be changed until the tube is in resonance, at which point an increase in the sound intensity can be heard. In an experiment, the speaker is driven at a fixed frequency of 1080 Hz, and three resonances are observed when the water level is at distances of $x_1 = 6.5$ cm, $x_2 = 22.2$ cm, and $x_3 = 37.7$ cm below the top of the tube. Find the value of the speed of sound from these data.

Solution The air column acts like a tube of variable length closed at one end. The pattern of standing waves shows a pressure node near the speaker and a pressure antinode at the surface of the water. Since we do not know the end correction, we cannot use the given data directly to find the speed of sound from Eq. 19-28. However, we note from the resonance conditions shown in Fig. 19-7b that the distance between adjacent pressure nodes is $\frac{1}{2}\lambda$; the same is true for the distance between adjacent antinodes. From the

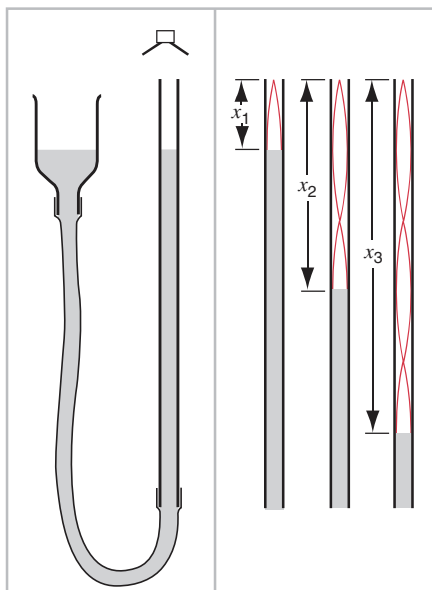


FIGURE 19-8. Sample Problem 19-4. An apparatus for measuring the speed of sound in air. The water level can be adjusted by raising and lowering the reservoir on the left, which is connected through a hose to the tube. At right are shown the pressure waveforms of the first three resonant modes for a fixed wavelength.

data given, we therefore conclude from the first two resonances that

$$\frac{1}{2}\lambda = x_2 - x_1 = 22.2 \text{ cm} - 6.5 \text{ cm} = 15.7 \text{ cm},$$

and similarly, from the second and third resonances,

$$\frac{1}{2}\lambda = x_3 - x_2 = 37.7 \text{ cm} - 22.2 \text{ cm} = 15.5 \text{ cm}.$$

The average of these two values, which we take as the best value from this measurement, is 15.6 cm, corresponding to a wavelength of $2(15.6 \text{ cm}) = 31.2 \text{ cm} = 0.312 \text{ m}$. We therefore deduce the speed of sound to be

$$v = \lambda f = (0.312 \text{ m})(1080 \text{ Hz}) = 337 \text{ m/s}.$$

Other than the end correction, what physical factors in this experiment (including the properties of the air) might influence the measured value?

19-7 VIBRATING SYSTEMS AND SOURCES OF SOUND*

A vibrating system transmits a wave through the air to the ears of the listener. This is the basic principle of the production of sound by voice or by musical instruments. We have already studied the propagation of the sound wave, and now to understand the nature of the sound we must study the vibrating system that produces it.

* For a listing of references on the physics of musical instruments and related topics, see "Resource Letter MA-2: Musical Acoustics," by Thomas D. Rossing, *American Journal of Physics*, July 1987, p. 589.

As we discussed in Section 18-10 in the case of the vibrating string and in the previous section in the case of a column of air, a distributed system has a large (perhaps infinite) number of natural vibrational frequencies. These are the frequencies at which it *can* vibrate. Which of the frequencies *will* be present in the vibration depends on how the system is set into vibration.

Suppose the system is capable of vibrating at a number of frequencies $f_1, f_2, f_3, \dots$. We write these in ascending order, so that $f_1 < f_2 < f_3 < \dots$. The lowest frequency, f_1 , is called the *fundamental* frequency, and the corresponding mode of oscillation is called the *fundamental mode*. The higher frequencies are called *overtones*, with f_2 being the first overtone, f_3 the second overtone, and so on.

In certain systems, the overtones are all integer multiples of the fundamental:

$$f_n = n f_1, \quad (19-29)$$

where n is an integer. In such a case, the overtones are called *harmonics*. The first member of a harmonic sequence is the fundamental, the second harmonic is the first overtone, and so on.

Why do some vibrating systems produce pleasant sounds while others produce harsh or discordant sounds? When several frequencies are heard simultaneously, a pleasant sensation results if the frequencies are in the ratio of small whole numbers, such as 3:2 or 5:4. If a system produces overtones that are harmonics, its vibrations will include frequencies that have these ratios, and it would produce a pleasing sound. If the overtones are not harmonics, it is likely that the sound will be discordant. Much of the effort in the design of musical instruments is devoted to the production of harmonic sequences of overtones. Some instruments, such as those based on vibrating strings, produce overtones that are automatically harmonics when the vibrations have small amplitude. In other cases, the shape of the instrument must be carefully designed to make it harmonic; a bell is an example of such an instrument. The harmonics that an instrument produces give it its richness and diversity of tone, and they are critical to the beauty of the sound of the instrument. If instruments produced only fundamentals, they would all sound exactly alike.

We can classify musical instruments into three categories: those based on vibrating strings, those based on vibrating columns of air, and more complex systems including vibrating plates, rods, and membranes.

Vibrating Strings

These instruments include the bowed strings (violins, for example), plucked strings (guitar, harpsichord), and struck strings (piano).

If a string fixed at both ends is bowed, struck, or plucked, transverse vibrations travel along the string; these disturbances are reflected at the fixed ends, and a standing

wave pattern is formed. The natural modes of vibration of the string are excited, and these vibrations give rise to longitudinal waves in the surrounding air, which transmits them to our ears as a musical sound.

We have seen (Section 18-10) that a string of length L , fixed at both ends, can resonate at frequencies given by Eq. 18-46:

$$f_n = n \frac{v}{2L}, \quad n = 1, 2, 3, \dots \quad (19-30)$$

Here v is the speed in the string of the transverse traveling waves whose superposition can be thought of as giving rise to the vibrations; the speed v ($= \sqrt{F/\mu}$) is the same for all frequencies. (Note that v is *not* the speed of sound in air; even though Eq. 19-30 looks exactly like Eq. 19-26, v stands for different quantities in the two equations.) At any one of these frequencies the string contains a whole number n of loops between its ends; it has nodes at each end and $n - 1$ additional nodes equally spaced along its length (Fig. 19-9).

If the string is initially distorted so that its shape is the same as *any one* of the possible harmonics, it vibrates only at the frequency of that particular harmonic. The initial conditions usually arise from striking or bowing the string, however, and in such cases not only the fundamental but many of the overtones are present in the resulting vibration. We have a superposition of several natural modes of oscillation. The actual displacement is the sum of the several harmonics with various amplitudes. The impulses that are sent through the air to the ear and brain give rise to one net effect, which is characteristic of the particular stringed instrument. The quality of the sound of a note of any particular frequency played by an instrument is determined by the number of overtones present and their respective intensities. Figure 19-10 shows the sound spectra and corresponding waveforms for the violin and piano.

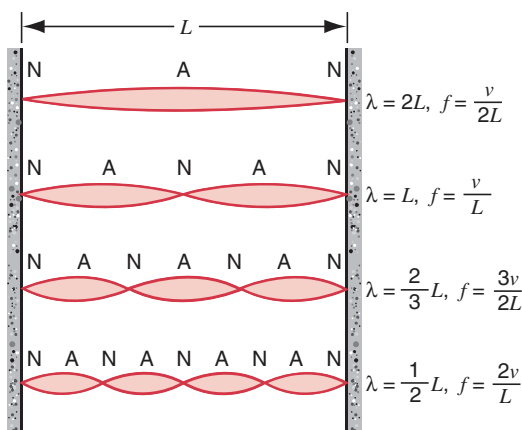


FIGURE 19-9. The first four resonant modes of a vibrating string fixed at both ends. Nodes and antinodes of displacement are denoted by N and A.

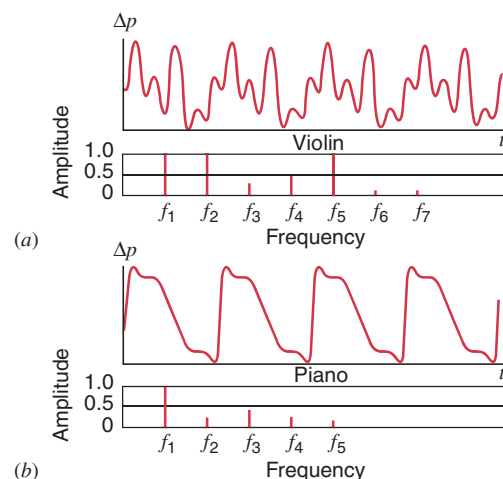


FIGURE 19-10. Waveforms and sound spectra for two stringed instruments, (a) violin and (b) piano, each playing a note of fundamental frequency $f_1 = 440$ Hz (concert A). The sound spectrum below each waveform shows the harmonics that are present in the complex tone and their relative amplitudes.

Vibrating Air Columns

An organ pipe is a simple example of a sound originating in a vibrating air column. If both ends of a pipe are open and a stream of air is directed against an edge at one end, standing longitudinal waves can be set up in the tube. The air column then resonates at its natural frequencies of vibration, given by Eq. 19-26. As with the bowed string, the fundamental and the overtones (which are harmonics) are produced at the same time. If one end of the tube is closed, the fundamental frequency is reduced by half, relative to its value for an open tube of the same length, and only the odd harmonics are present, which changes the quality of the sound. That is, an open pipe produces the same fundamental tone as a closed pipe of half the length, but because the mixture of harmonics is different in the two pipes, the quality of the tones differs.

Reed instruments, such as the clarinet, produce their tones differently. Air is forced through a narrow opening, one side of which is covered by a reed that has elastic properties. According to Bernoulli's equation, the high-speed air passing through the narrow opening causes a local region of low pressure inside the mouthpiece. The outside pressure exceeds the inside pressure, which forces the reed inward so that it covers the opening. As soon as the opening is covered, the air flow is interrupted, the dynamic low-pressure region is eliminated, and the reed pops open, allowing the air flow to start again. This repeated opening and closing of the air passage causes maximum variations in pressure at the mouthpiece end of the instrument, which therefore behaves like an antinode of pressure. In a clarinet, the other end of the instrument is open, and therefore the resonances of the instrument are those given by Eq. 19-28 for a tube closed at one end and open at the other. Some wind instruments, such as the flute, use a method similar to the organ

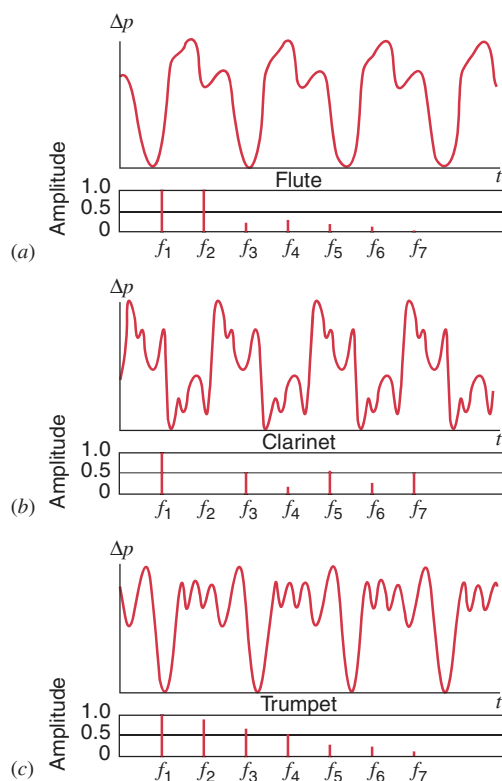


FIGURE 19-11. Waveforms of some wind instruments: (a) flute, (b) clarinet, and (c) trumpet, and their sound spectra, as in Fig. 19-10. Note that the clarinet spectrum shows mostly the odd harmonics, whereas the flute and trumpet have odd as well as even harmonics.

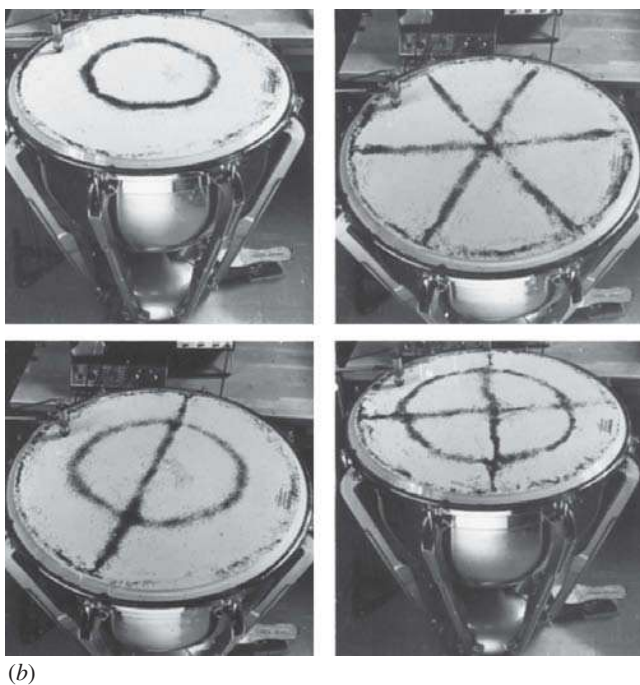
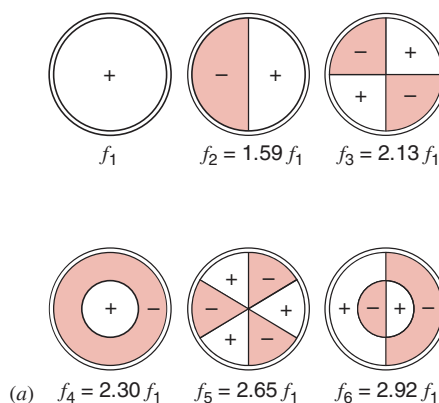
pipe to produce the tone, such that the mouthpiece behaves like an open end; their resonant frequencies are given by Eq. 19-26. Still others (oboe, saxophone), which use a reed to produce their tone, have a conical (that is, tapered) rather than a cylindrical bore, which gives them overtones that are approximately harmonics, odd as well as even. The brass instruments (trumpet or trombone, for example) are also called *lip reed* instruments, because the player's lip acts like a reed, but again the bore is slightly tapered, and as a result the overtones contain all the harmonics. Figure 19-11 shows waveforms of some wind instruments.

Other Vibrating Systems

Vibrating rods, plates, and stretched membranes also give rise to sound waves. Consider a stretched flexible membrane, such as a drumhead. If it is struck a blow, a two-dimensional pulse travels outward from the struck point and is reflected again and again at the boundary of the membrane. If some point of the membrane is forced to vibrate periodically, continuous trains of waves travel out along the membrane. Just as in the one-dimensional case of the string, so here too standing waves can be set up in the two-dimensional membrane. Each of these standing waves has a certain frequency natural to (or characteristic of) the membrane. Again the lowest frequency is called the funda-

mental, and the others are overtones. Generally, many overtones are present along with the fundamental when the membrane is vibrating. These vibrations may excite sound waves of the same frequency.

The nodes of a vibrating membrane are lines rather than points (as in a vibrating string) or planes (as in a tube). Since the boundary of the membrane is fixed, it must be a nodal line. For a circular membrane fixed at its edge, possible modes of vibration together with their nodal lines are shown in Fig. 19-12. The natural frequency of each mode is



(b)

FIGURE 19-12. (a) The lowest six resonant modes of a circular drumhead clamped at its rim. The lines represent nodes; the rim is also a nodal line. The + or - signs indicate that, at a given instant, a particular region is moving up out of the page or down into the page. In this case the overtones are not integral multiples of the fundamental and are thus not harmonics. (b) The vibrational patterns of a kettledrum in the modes numbered 4, 5, and 6, and one additional mode not illustrated in (a). They are made visible by sprinkling dark powder on the drumhead and setting it into vibration at the proper frequency using a mechanical vibrator. As the drumhead vibrates, the powder is shaken and eventually settles on the nodal lines, where there is no motion.

given in terms of the fundamental f_1 . The frequencies of the overtones are not harmonics; that is, they are not integral multiples of f_1 . Vibrating rods also have a nonharmonic set of natural frequencies. Rods and plates have limited use as musical instruments for this reason. In instruments such as the xylophone and the marimba, small bars of wood or metal are set into vibration by striking. The shape of the bars is carefully modified, making them thinner in the center, in such a way that the overtones become approximately harmonics.

19-8 BEATS

We have previously considered the effect of waves that are superimposed to give regions of maximum and minimum (zero) intensity, such as in the case of a standing wave in a tube. This illustrates a type of interference that we can call *interference in space*.

The same principle of superposition leads us to another type of interference, which we can call *interference in time*. In this case we examine the superposition of two waves at a given point as a function of the time. This superposition, which in general can result in quite complex waveforms, takes a particularly simple form when the two waves have nearly the same frequency. With sound such a condition exists when, for example, two instruments or two guitar strings are being tuned to one another.

Consider a point in space through which the waves are passing. Figure 19-13a shows the pressure produced at that point by the two waves separately as a function of time. For simplicity we have assumed that the two waves have equal amplitude, although this is not necessary. The resultant pressure at that point as a function of time is the sum of the individual pressures and is plotted in Fig. 19-13b. We see that the amplitude of the resultant wave is not constant but

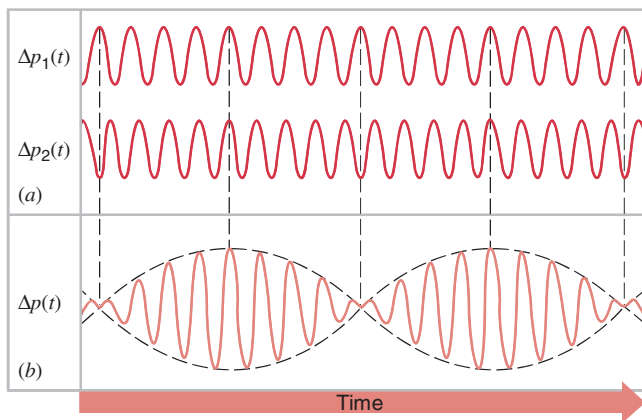


FIGURE 19-13. (a) Two sinusoidal waveforms of nearly equal frequencies. (b) The superposition of the two waveforms. Note that the two waves in part (a) go from being in phase, giving a resultant of large amplitude, to being out of phase, giving a resultant of zero amplitude. The dashed curves show the sinusoidal variation of the modulating envelope with angular frequency ω_{amp} .

varies with time. In the case of sound the varying amplitude gives rise to variations in loudness, which are called *beats*.

Let us represent the variation in pressure with time (for constant x) produced by one wave as

$$\Delta p_1(t) = \Delta p_m \sin \omega_1 t,$$

where we have chosen the phase constant to enable us to write the wave in this simple form. The pressure variation at the same point produced by the other wave of equal amplitude is represented as

$$\Delta p_2(t) = \Delta p_m \sin \omega_2 t.$$

By the superposition principle, the resultant pressure is

$$\begin{aligned} \Delta p(t) &= \Delta p_1(t) + \Delta p_2(t) \\ &= \Delta p_m (\sin \omega_1 t + \sin \omega_2 t). \end{aligned} \quad (19-31)$$

Using the trigonometric identity

$$\sin A + \sin B = 2 \cos \frac{A - B}{2} \sin \frac{A + B}{2},$$

Eq. 19-31 can be written

$$\Delta p(t) = \left[2\Delta p_m \cos \left(\frac{\omega_1 - \omega_2}{2} t \right) \right] \sin \left(\frac{\omega_1 + \omega_2}{2} t \right). \quad (19-32)$$

So far everything we have done applies to any two waves, no matter what their frequencies. When the frequencies are nearly the same, Eq. 19-32 can be simplified by writing the second factor in terms of the average angular frequency ω_{av} of the two waves,

$$\omega_{\text{av}} = \frac{\omega_1 + \omega_2}{2}. \quad (19-33)$$

The first factor, contained in the brackets of Eq. 19-32, gives a time-varying amplitude to the sinusoidal variation of the second factor. This amplitude factor varies with an angular frequency

$$\omega_{\text{amp}} = \frac{|\omega_1 - \omega_2|}{2}. \quad (19-34)$$

In terms of ω_{av} and ω_{amp} , we can write Eq. 19-32 as

$$\Delta p(t) = [2\Delta p_m \cos \omega_{\text{amp}} t] \sin \omega_{\text{av}} t. \quad (19-35)$$

If ω_1 and ω_2 are nearly equal, the amplitude frequency ω_{amp} is small, and the amplitude fluctuates slowly. Figure 19-13 shows the superposition of the two waves according to Eq. 19-32. Notice that in the case of nearly equal frequencies, the rapid variation of the resultant wave occurs with a frequency that is approximately that of either of the two added waves. The overall amplitude of the resultant varies slowly with the amplitude frequency ω_{amp} , which defines an “envelope” within which the more rapid variation occurs. This phenomenon is a form of *amplitude modulation*, which has a counterpart (side bands) in AM radio receivers.

In the case shown in Fig. 19-13b, the ear would perceive a tone at a frequency $f_{\text{av}} (= \omega_{\text{av}}/2\pi)$ which is approxi-

mately the same as the frequencies f_1 ($= \omega_1/2\pi$) or f_2 ($= \omega_2/2\pi$) of the two component waves. The tone grows alternately loud and soft as the amplitude of the resultant varies with time, having maxima and minima as shown in Fig. 19-13b.

A beat—that is, a maximum of intensity—occurs whenever $\cos \omega_{\text{amp}} t$ equals $+1$ or -1 , since the intensity depends on the *square* of the amplitude. Each of these values occurs once in each cycle of the envelope (see Fig. 19-13), so the number of beats per second is twice the number of cycles per second of the envelope. The beat angular frequency ω_{beat} is then

$$\omega_{\text{beat}} = 2\omega_{\text{amp}} = |\omega_1 - \omega_2|. \quad (19-36)$$

Using $\omega = 2\pi f$, we can rewrite this expression as

$$f_{\text{beat}} = |f_1 - f_2|. \quad (19-37)$$

Hence *the number of beats per second equals the difference of the frequencies of the component waves*. Beats between two tones can be detected by the ear up to a frequency of about 15 Hz. At higher frequencies individual beats cannot be distinguished in the sound produced. Musicians often listen for beats when tuning certain instruments. The tuning is changed until the beat frequency decreases and the beats disappear.

SAMPLE PROBLEM 19-5. A violin string that should be tuned to concert A (440 Hz) is slightly mistuned. When the violin string is played in its fundamental mode along with a concert A tuning fork, 3 beats per second are heard. (a) What are the possible values of the fundamental frequency of the string? (b) Suppose the string were played in its first overtone simultaneously with a tuning fork one octave above concert A (880 Hz). How many beats per second would be heard? (c) When the tension of the string is increased slightly, the number of beats per second in the fundamental mode increases. What was the original frequency of the fundamental?

Solution (a) From Eq. 19-37, we know that the frequency f_1 of the string differs by the beat frequency (3 Hz) from the frequency f_2 of the tuning fork (440 Hz), but from the number of beats per second alone, we cannot tell whether the string has a higher or lower frequency. Thus the possible frequencies are

$$f_1 = 440 \text{ Hz} \pm 3 \text{ Hz} = 443 \text{ Hz} \quad \text{or} \quad 437 \text{ Hz}.$$

(b) In the first overtone, the frequency of the string is twice its fundamental frequency, and thus either 886 Hz or 874 Hz. When played against a 880-Hz tuning fork, the frequency difference in either case is 6 Hz, and thus 6 beats per second would be heard.

(c) Increasing the tension in the string raises the speed of transverse waves and therefore raises the fundamental frequency (see Eq. 19-30). Since we are given that this increases the beat frequency, we conclude that the frequency of the fundamental mode was previously greater than 440 Hz, since increasing the frequency made the difference from 440 Hz even greater. Thus the string was originally tuned to 443 Hz, and to bring it into proper tuning the tension must be reduced.

19-9 THE DOPPLER EFFECT

When a listener is in motion toward a stationary source of sound, the pitch (frequency) of the sound heard is higher than when the listener is at rest. If the listener is in motion away from the stationary source, a lower pitch is heard. We obtain similar results when the source is in motion toward or away from a stationary listener. The pitch of the whistle of a locomotive or the siren of a fire engine is higher when the source is approaching the hearer than when it has passed and is receding.

In a paper written in 1842, Christian Johann Doppler (1803–1853, Austrian) called attention to the fact that the color of a luminous body must be changed by relative motion of the body and the observer. This *Doppler effect*, as it is called, applies to waves in general. Doppler himself mentions the application of his principle to sound waves. An experimental test was carried out in Holland in 1845 by Buys Ballot, “using a locomotive drawing an open car with several trumpeters.”

Moving Observer, Source at Rest

We now consider the Doppler effect for sound waves, treating only the special case in which the source and observer move along the line joining them. Let us adopt a reference frame at rest in the medium through which the sound travels. Figure 19-14 shows a source of sound S at rest in this frame and an observer O moving toward the source at a speed v_o . The circles represent wavefronts, spaced one wavelength apart, traveling through the medium. An observer at rest in the medium would receive vt/λ waves in time t , where v is the speed of sound in the medium and λ is the wavelength. Because of the motion toward the source, however, the observer receives $v_o t/\lambda$ additional waves in

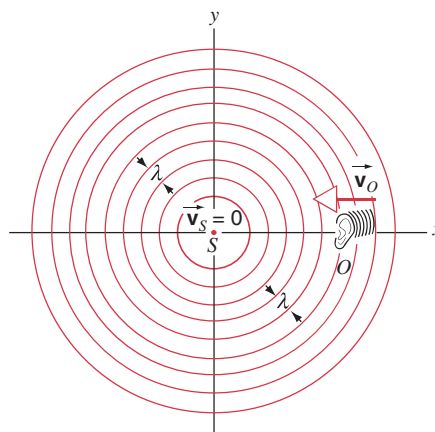


FIGURE 19-14. A stationary source of sound S emits spherical wavefronts, shown one wavelength apart. An observer O , represented by the ear, moves with speed v_o toward the source. The moving observer encounters more waves per second than an observer at rest and therefore measures a higher frequency. The observer would measure a *lower* frequency for motion away from the source.

this same time t . The frequency f' that is actually heard is the number of waves received per unit time, or

$$f' = \frac{vt/\lambda + v_O t/\lambda}{t} = \frac{v + v_O}{\lambda} = \frac{v + v_O}{v/f}.$$

That is,

$$f' = f \frac{v + v_O}{v} = f \left(1 + \frac{v_O}{v} \right). \quad (19-38)$$

The frequency f' heard by the observer is the frequency f heard at rest plus the increase $f(v_O/v)$ arising from the motion of the observer. When the observer is in motion *away* from the stationary source, there is a *decrease* in frequency $f(v_O/v)$ corresponding to the waves that do not reach the observer in each unit of time because of the receding motion. Then

$$f' = f \frac{v - v_O}{v} = f \left(1 - \frac{v_O}{v} \right). \quad (19-39)$$

Hence the general relation holding when the *source is at rest* with respect to the medium but the *observer is moving* through it is

$$f' = f \frac{v \pm v_O}{v}, \quad (19-40)$$

where the plus sign holds for motion toward the source and the minus sign holds for motion away from the source. Note that the change in frequency occurs because the observer intercepts more or fewer waves each second as a result of the motion through the medium.

Moving Source, Observer at Rest

When the source is in motion *toward* a stationary observer, the effect is a shortening of the wavelength (see Fig. 19-15), because the source is following after the approaching

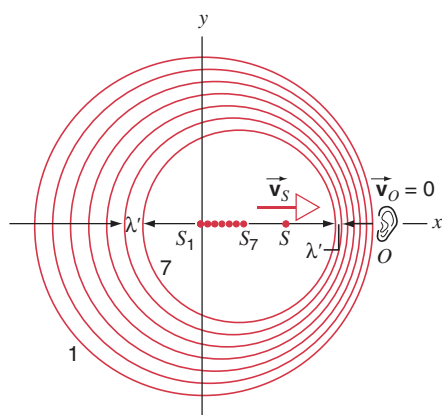


FIGURE 19-15. Here the observer O is at rest, with the source moving toward it at speed v_s . Wavefront 1 was emitted when the source was at S_1 , wavefront 7 when the source was at S_7 , and so on. At the instant of this drawing, the source is at S . The observer measures a shorter wavelength because of the “bunching up” of the wavefronts along the motion. An observer on the negative x axis, from whom the source would be moving away, would measure a longer wavelength.

waves, and the crests therefore come closer together. If the frequency of the source is f and its speed is v_s , then during each vibration it travels a distance v_s/f , and each wavelength is shortened by this amount. Hence the wavelength of the sound arriving at the observer is not $\lambda = v/f$ but $\lambda' = v/f - v_s/f$. The frequency of the sound heard by the observer is increased and is given by

$$f' = \frac{v}{\lambda'} = \frac{v}{(v - v_s)/f} = f \frac{v}{v - v_s}. \quad (19-41)$$

If the source moves *away* from the observer, the wavelength emitted is v_s/f greater than λ , so that the observer hears a decreased frequency—namely,

$$f' = \frac{v}{(v + v_s)/f} = f \frac{v}{v + v_s}. \quad (19-42)$$

Hence the general relation holding when the *observer is at rest* with respect to the medium but the *source is moving* through it is

$$f' = f \frac{v}{v \pm v_s}, \quad (19-43)$$

where the minus sign holds for motion toward the observer and the plus sign holds for motion away from the observer. Note that the change here is the shortening or increasing of the wavelength transmitted through the medium due to the motion of the source through the medium.

If *both source and observer move* through the transmitting medium, you should be able to show that the observer hears a frequency

$$f' = f \frac{v \pm v_O}{v \mp v_s}, \quad (19-44)$$

where the upper signs (+ numerator, − denominator) correspond to the source and observer moving along the line joining the two in the direction toward the other, and the lower signs in the direction away from the other. Equation 19-44 incorporates all four different possibilities, as Sample Problem 19-6 shows. Note that Eq. 19-44 reduces to Eq. 19-40 when $v_s = 0$ and Eq. 19-43 when $v_O = 0$, as it must.

If a source of sound is moved away from an observer and toward a wall, the observer hears two notes of different frequency. The note heard directly from the receding source is lowered in pitch by the motion. The other note is due to the waves reflected from the wall, and this is raised in pitch (because the source is moving *toward* the wall, and the wall “hears” the higher frequency). The superposition of these two wave trains produces beats. A similar effect occurs if a wave from a stationary source is reflected from a moving object. The beat frequency can be used to deduce the speed of the object. This is the basic principle of radar speed monitors, and it is also used to track satellites.

The discussion in this section applies to the Doppler shift for sound waves and other similar mechanical waves. Light waves also show the Doppler effect; however, because there is no medium of propagation for light, the for-

mulas developed in this section do not apply. See Chapter 39 for a discussion of the Doppler effect for light waves.

SAMPLE PROBLEM 19-6. The siren of a police car emits a pure tone at a frequency of 1125 Hz. Find the frequency that you would perceive in your car under the following circumstances: (a) your car at rest, police car moving toward you at 29 m/s (65 mi/h); (b) police car at rest, your car moving toward it at 29 m/s; (c) you and the police car moving toward one another at 14.5 m/s; (d) you moving at 9 m/s, police car chasing behind you at 38 m/s.

Solution All four parts of this problem can be solved using Eq. 9-44. (a) Here $v_O = 0$ (your car is at rest) and $v_S = 29$ m/s. We choose the upper (minus) sign in the denominator of Eq. 19-44, because the police car is moving toward you. We thus obtain, using $v = 343$ m/s for the speed of sound in still air,

$$f' = f \frac{v}{v - v_S} = (1125 \text{ Hz}) \frac{343 \text{ m/s}}{343 \text{ m/s} - 29 \text{ m/s}} = 1229 \text{ Hz}.$$

(b) In this case $v_S = 0$ (the police car is at rest) and $v_O = 29$ m/s. We choose the upper (plus) sign in the numerator of Eq. 19-44, because you are moving toward the police car, and we find

$$f' = f \frac{v + v_O}{v} = (1125 \text{ Hz}) \frac{343 \text{ m/s} + 29 \text{ m/s}}{343 \text{ m/s}} = 1220 \text{ Hz}.$$

(c) In this case $v_S = 14.5$ m/s and $v_O = 14.5$ m/s. We choose the upper signs in both the numerator and denominator of Eq. 19-44, because you and the police car are moving toward each other. We thus obtain

$$f' = f \frac{v + v_O}{v - v_S} = (1125 \text{ Hz}) \frac{343 \text{ m/s} + 14.5 \text{ m/s}}{343 \text{ m/s} - 14.5 \text{ m/s}} = 1224 \text{ Hz}.$$

(d) Here $v_O = 9$ m/s and $v_S = 38$ m/s. You are moving away from the police car, so we choose the lower (minus) sign in the numerator, but the police car is moving *toward* you, so we choose the upper (minus) sign in the denominator. The result is

$$f' = f \frac{v - v_O}{v - v_S} = (1125 \text{ Hz}) \frac{343 \text{ m/s} - 9 \text{ m/s}}{343 \text{ m/s} - 38 \text{ m/s}} = 1232 \text{ Hz}.$$

Note that in all four cases in this sample problem, the relative speed between you and the police car is the same—namely, 29 m/s—but the perceived frequencies are different in the four cases. The Doppler shift for sound is determined not only by the relative speed between source and observer, but also by both of their speeds relative to the medium that carries the sound.

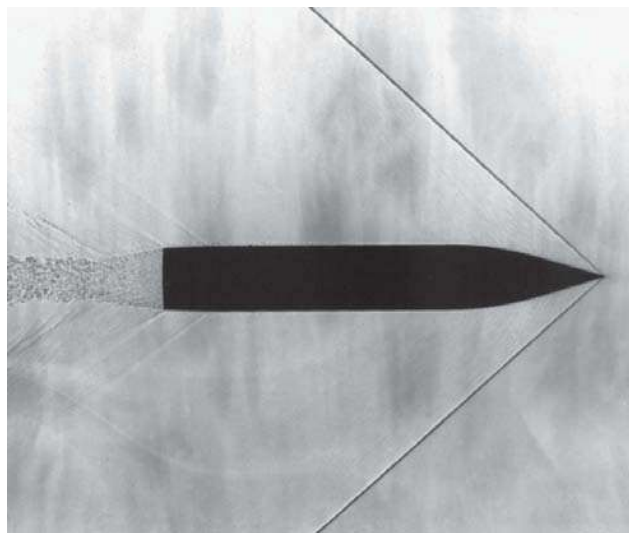
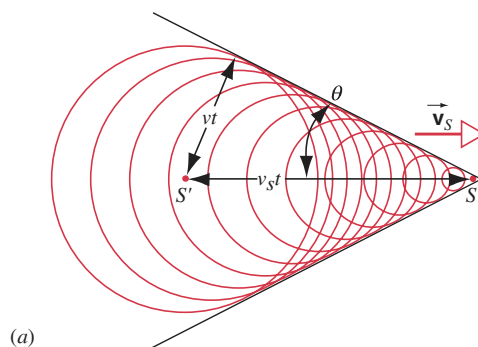
Effects at High Speed (Optional)

When v_O or v_S becomes comparable in magnitude to v , the formulas just given for the Doppler effect usually must be modified. One modification is required because the linear relation between restoring force and displacement assumed up until now may no longer hold in the medium. The speed of wave propagation is then no longer the normal phase speed, and the wave shapes change in time. Components of the motion at right angles to the line joining source and observer also contribute to the Doppler effect at these high

speeds. When v_O or v_S exceeds v , the Doppler formula does not apply; for example, if $v_S > v$, the source gets ahead of the wave in one direction; if $v_O > v$ and the observer moves away from the source, the wave never catches up with the observer.

There are many instances in which the source moves through a medium at a speed greater than the phase speed of the wave in that medium. In such cases the wavefront takes the shape of a cone with the moving body at its apex. Some examples are the bow wave from a speedboat on the water and the “shock wave” from an airplane or projectile moving through the air at a speed greater than the speed of sound in that medium (supersonic speeds). Another example is the so-called Cerenkov radiation, which consists of light waves emitted by charged particles that move through a medium with a speed greater than the phase speed of light in that medium. The blue glow of the water that often surrounds the core of a nuclear reactor is one type of Cerenkov radiation.

In Fig. 19-16a we show the present positions of the spherical waves that originated at various positions of a source during its motion. The radius of each sphere at this



(b)

FIGURE 19-16. (a) Wavefronts of a source moving at supersonic speed. The wavefronts are spherical and their envelope is a cone. Compare this figure with Fig. 19-15. (b) A photograph of a projectile fired from a gun at Mach 2. Note the Mach cone.

time is the product of the wave speed v and the time t that has elapsed since the source was at its center. The envelope of these waves is a cone whose surface makes an angle θ with the direction of motion of the source. From the figure we obtain the result

$$\sin \theta = \frac{v}{v_s}. \quad (19-45)$$

For surface water waves the cone reduces to a pair of intersecting lines. In aerodynamics the ratio v_s/v is called the *Mach number*. An aircraft flying at supersonic speed generates a *Mach cone* similar to that shown in Fig. 19-16. When

the edge of that cone intercepts the ground below, we hear a “sonic boom,” which (contrary to common belief) is *not* associated with an aircraft “breaking the sound barrier.” The sonic boom is merely the total effect of the concentration on one surface of the aircraft’s radiated sound energy, which would normally radiate in all directions at subsonic speeds. As the photograph of Fig. 19-16*b* shows, it might be possible to hear two sonic booms from the same aircraft, one from the leading edge and another from the trailing edge. (Note also that the Mach cone never intercepts the projectile itself; thus the aircraft’s passengers do not hear the sonic boom.) ■

MULTIPLE CHOICE

19-1 Properties of Sound Waves

19-2 Traveling Sound Waves

- Which is larger for a sound wave in a fluid, the relative density variations, $\Delta\rho_m/\rho_0$, or the relative pressure variations, $\Delta p_m/p_0$?
 (A) $\Delta\rho_m/\rho_0 > \Delta p_m/p_0$, always.
 (B) $\Delta\rho_m/\rho_0 = \Delta p_m/p_0$, always.
 (C) $\Delta\rho_m/\rho_0 < \Delta p_m/p_0$, always.
 (D) Which is larger varies, depending on the pressure and the bulk modulus.

19-3 The Speed of Sound

- A thin steel rod of length 12 km is suspended in a frictionless tube. A researcher gently taps one end with a hammer. The researcher on the other end hears the tap.
 (A) at the same instant.
 (B) almost, but not quite, instantaneously.
 (C) approximately 2 seconds later.
 (D) approximately 30 seconds later.
 (E) approximately 1/2 minute later.
- Which is larger, the velocity of a sound wave v or the amplitude of the velocity variations u_m of the oscillating sound particles?
 (A) v is always greater than u_m .
 (B) v and u_m are equal.
 (C) v must be less than u_m .
 (D) The two velocities are unrelated to each other.

19-4 Power and Intensity of Sound Waves

- Spherical sound waves are emitted uniformly in all directions from a point source. The variation in sound level SL as a function of distance r from the source can be written as
 (A) $SL = -b \log r^a$
 (B) $SL = a - b (\log r)^2$
 (C) $SL = a - b \log r$
 (D) $SL = a - b/r^2$
 where a and b are positive constants.
- If the average power of a sound wave is expressed in terms of displacement amplitudes s_m and frequencies f , then
 (A) $P_{av} \propto f^2 s_m^2$. (B) $P_{av} \propto f s_m^2$.
 (C) $P_{av} \propto f^{-1} s_m^2$. (D) $P_{av} \propto f^{-2} s_m^2$.

19-5 Interference of Sound Waves

- One way to improve the performance of bass speakers on a sound system is to attach a curved tube to the back of the speaker that passes near the front so that the sound waves from the back of the speaker are allowed to interfere constructively with the sound waves from the front (Fig. 19-17). If the average wavelength of the sound coming from the speaker is λ , then the length of the tube to produce constructive interference should be
 (A) $\lambda/4$. (B) $\lambda/2$. (C) $3\lambda/4$. (D) λ .



FIGURE 19-17. Multiple-choice question 6.

19-6 Standing Longitudinal Waves

- What is the pattern for resonant frequencies of a tube closed at both ends?
 (A) The same as a tube open at both ends, $f_n = nv/2L$, $n = 1, 2, 3, 4, 5, \dots$
 (B) The same as a tube closed at one end, $f_n = nv/4L$, $n = 1, 3, 5, 7, 9, \dots$
 (C) $f_n = nv/8L$, $n = 1, 5, 9, 13, 17, \dots$
 (D) A tube closed at both ends does not have any resonant frequencies.

19-7 Vibrating Systems and Sources of Sound

- The octave thumb key on a clarinet forces the resonance mode from the fundamental to the first overtone. Pressing the